The Evaluation of Nonlinear Musical Structures

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Our experience of time through music is diverse and often irregular. In Western and non-Western music, composers have explored the idea of exploiting this relationship to create structures that destabilize, fragment and even suspend time.

A definition of Nonlinear Structure is proposed based upon evaluation of the level of integration, contingency, compressibility and determinacy of a work. Exemplar works by composers Earle Brown, Béla Bartók, Olivier Messiaen and Brian Eno are discussed.

Introduction

Over the last century, the term nonlinear has been applied to the structure of a broad range of musical works. Arguably the emergence of nonlinearity as a structural technique was a logical consequence of the increasing complexity of formal experimentation during the Modernist era. The frequent application of terms that imply disruption of linearity such as interruption, reordering, disjunction, fragmentation, juxtaposition, permutation, and stratification in discussion of work by composers in the early years of the twentieth century such as Mahler, Ives, Debussy and Stravinsky are indicators of the increasing prevalence of nonlinear aesthetic.

The idea of nonlinear musical formal structure might appear, at face value, to be something of a contradiction in terms. As a time-based medium, music incontestably unfolds moment by moment upon a linear canvas. Musical structure, however, is evaluated through the comparison of a succession of musical events. The evaluation of structure presupposes that the succession of sounds comprising a musical work is capable of schematisation – of reduction to a simplified representation. The potential results of this process form a continuum of states of structural complexity, bounded by completely structured linearity and complete unstructured nonlinearity.

This discussion argues that Nonlinear structures occupy the middle ground between these poles: resisting the formation of a unified directionality, but retaining at least a minimal relationship between at some of their components.

This chapter seeks both to identify the circumstances that lead to the emergence of nonlinearity in music, and to explore and classify the range of manifestations of what might be considered from an analytical perspective, to be Nonlinear structures. This goal is independent of the consideration of techniques and methodologies utilized by composers to generate musical material. Although composers have developed compositional techniques that involve nonlinear or potentially nonlinear processes such as chance, “automatism”, probability and “found systems”, such approaches may not give rise to nonlinear formal structures.

An example is Musikalisches Würfelspiel (1792) set of rules and musical materials allowing minuets to be (literally) cast according to coin tosses. This work, attributed to Mozart (O’Beirne 1967), is frequently cited as a precursor both to aleatoric music and algorithmic music. The indeterminate nature of choices delivers works that are analytically indiscernible from minuets composed in the traditional, supposedly determinate, manner (this is of course the paradoxical diversion of the “game”).

Here the syntax of the tonal structures necessary to produce a minuet are sufficiently simple that a large number of possible “solutions” are admissible to maintain a convincing continuity, so that any choice will yield an acceptable outcome.

The material is arranged in the charts in such a way that all compositional problems such as cadences are automatically adjusted and the compositional process is reduced to a game of throwing dice and matching measure numbers.
By using repeat signs and a chart for the B material, a composition eighty measures in length is generated, having the form AABB aabb AB.

(Husarik 1983 p. 7)

The compositional technique employed in creating a Musikalisches Würfelspiel minuet is nonlinear according to Kramer’s definition that “the generation of each event (is) independent of all others” (Kramer, 1981 p. 554). The fact that it is still capable of producing a linear formal structure underlines the independence of compositional methods and formal outcomes. In the dice game minuet, linearity is clearly provided by the high degree of continuity between its musical components.

In 1969 Mozart’s compositional techniques were revisited in an altogether different context in John Cage and Lejaren Hiller’s multimedia work HPSCHD. A computing student Edward Kobrin (Cage and Kostelanetz 1988 p. 41) created a program called DICEGAME algorithmically encoding the Würfelspiel rules (Husarik 1983 p. 7). Using this program, Cage and Hiller generated seven harpsichord solos, substituting the original musical material provided by Mozart with selections from Mozart piano sonatas, “Beethoven's Appassionata Sonata; Chopin's Prelude in D Minor, opus 28; Schumann's "Reconnaissance" from Carnaval; Gottschalk's The Banjo, Busoni's Sonata no. 2; Cage's Winter Music; and Hiller’s Sonata no. 5’ that were chosen at random according to coin tosses (Heimbecker 2008 p. 493). Cage and Hiller’s realizations of the Würfelspiel, comprising musical materials that were not chosen according to any overarching continuity, would result in nonlinear structures, despite the ordering of their segments being generated using the same nonlinear process as the original Mozart version.

These examples point to the primary function of disjunction between musical substructures in the determination of nonlinearity in a formal structure. The following section explores the minimal circumstances in which nonlinearity emerges in a musical composition.

The Emergence of the Nonlinear Event

The emergence of substructures from the fabric of musical discourse derives from shifts in the level of continuity from one moment to the next. These shifts are marked by a weakening of the continuity of “form-bearing” musical parameters, that forms a musical discontinuity, between regions of relatively strong internal cohesion (see Figure 1).

The use of discontinuity to mark structural divisions is a common musical practice in musical composition. In Sonata Form, for example, continuity is often broken by disjunctions such as a quickening of harmonic rhythm, a cadence in a new key, and a short silence, to mark the boundary between substructures. However, a high level of homogeneity and contingency between the substructures is usually maintained through relative continuity in other parameters such as meter, tempo and proximity of the modulation.

When the degree of discontinuity between two substructures is raised to a critical point, the impression of homogeneity and stability of the musical material breaks down. Borrowing a term from music perception several critics have called this an absence of “belongingness” the two substructures cannot be reconciled with one another, as belonging to an unfolding continuity.

This occurrence – a nonlinear event - is the minimal level at which nonlinearity, if not yet nonlinear formal structure, can operate. In a nonlinear event, disjunction between substructures reaches a critical point and continuity cannot be maintained across the rupture. Pierre Boulez described this process as “anesthetizing the frontiers” between musical substructures, leading to a situation in which “listening time is no longer directional but time-bubbles, as it were” (Boulez 1986 p. 178).

The first issue is the degree to which the work is comprised of integrated musical materials or materials that may be divided into discrete disjunctive substructures. The emergence of nonlinear formal structure depends on the assessment of the relationships between these substructures. The presence of discrete and non-contingent substructures is the most important indicator of nonlinearity in a musical composition. The range of variables that can potentially contribute to the evaluation of Nonlinear Structure will be discussed in the following section.
The Evaluation of Non-Linearity

Strategies for representing musical structure are often hierarchical: they involve the reduction of larger spans of musical time into schematically represented substructures.

These issues, represented in Figure 2, situate nonlinear formal structure on the continuum of formal complexity, as epitomised by a high level of non-contingency and discreteness and intermediate levels of compressibility and determinacy.

The emergence of Nonlinear Structure from a continuum presumes the presence of boundaries at which linearity gives way to nonlinear structure and at which structure itself gives way. The following section explores several examples of such boundary works.

Defining the Boundaries of Nonlinear Structure

Earle Brown’s (1926-2002) work Event-Synergy II (1967) occupies a central position upon the continuum of formal complexity and as such is an example of a Nonlinear structure. In this work 19 performers are divided into two ensembles (designated A and B) each with an independent conductor (See Figure 3).

The conductors indicate to the performers via hand signals which of four “events” they are to play. The conductors spontaneously determine the order and duration of the events and indeed the duration of the work as a whole, during the performance. They may also freely cue other indications such as tempo, dynamic and fermata.

Figure 4 shows excerpts of the score from each of the four events, as well as the seating arrangement for the ensemble. Event 1 and Event 3 (examples a. and b. respectively) are further divisible: Event 1 into five sections and Event 3 into four subsections. These subsections are performable in any number or order.

The musical material in each of the four events is distinct. The parametrical variety of the events is summarized in Table 1. In addition to the parameters listed, the conductors are afforded significant not only in deciding the order and tempo of the events, but also to “over-ride” the indicated dynamic values and raise or lower the over-all loudness” (Brown 1962).
The spatial disposition of the ensembles provides a further distinction between the materials performed by each group. The discrete nature of the four Events ensures disjunction between materials in any particular iteration of the work. The level of integration and contingency in the work is therefore low.

Event-Synergy is an “open” work, in the sense that the final structure of any performance is primarily dependent upon the decisions taken by the conductors. The possible number of instantiations of the work, considering that it does not have a fixed duration, is extremely large within the boundaries set by the composer, namely the materials, the performers and the number of conductors. Despite the indeterminacy of these factors it is still feasible both to analyse a specific performance of the work and to schematically represent a notional realization of the work (see Figure 5). Given the prescribed nature of the materials, orchestration and the rule set governing the work, the compressibility and determinacy of Event-Synergy might be regarded as moderate in comparison to utter lack of structure.

**Figure 4: Score Excerpts from Earle Brown: Event – Synergy II (1967): A1 (Winds), b.) Event B3 (Strings), c.) Event B2 (Strings) d.) Seating Arrangements for the Ensembles A and B and e.) Event B4 (Strings)**
The example of Event-Synergy draws attention to the outer boundary of the continuum of formal complexity, between structured and unstructured nonlinearity. In defining this boundary, composer Brian Eno’s (1948– ) experience of listening to a recording of chance environmental events that were arbitrarily captured on tape, is pertinent. Although something of a “thought experiment” given that this “work” has never been publically available it focuses on the role of determinacy in evaluating nonlinearity.

According to Eno, he “recorded whatever sounds there happened to be: cars going by, dogs, people”, cut it down to three and a half minutes and then “kept running it over and over” (Toop, 1995 p. 129).

I tried to learn it, exactly as one would a piece of music: oh yeah, that car, accelerates the engine, the revs in the engine go up and then that dog barks, and then you hear that pigeon off to the side there. This was an extremely interesting thing to do, first of all because I found you can learn it. Something that is completely arbitrary and disconnected as that, with sufficient listenings, becomes highly connected.”

( Ibid)

In this example, three of the distinguishing features of Nonlinear structures integration, contingency and compressibility are presumably at or near their minimum threshold, however determinacy, in the form of a capability to repeat the structure very exactly, is maximized. The sounds alone might be considered as unstructured, but the high degree of determinacy of the recording of the sounds draws it within the boundary of structure.

This example emphasizes the important role that technology has played in the issue of nonlinearity: before the advent of recording this technique for capturing a segment of unstructured sound was impossible. It is also something of a perverse example, in that the obsessive listening strategy needed to render “completely arbitrary and disconnected” sonic events as “highly connected” is not a normative behaviour. The issue is pertinent as repetition is the only link such a work retains with the notion of structure. Increasing familiarity with the sonic materials promotes the formation of connection and contingency between the discrete events.

Theorist Holger Schulze comments on Eno’s observations:

“We simply cannot bear to be surrounded by anything that is literally meaningless and generated by chance. We forget its aleatoric genesis and find ourselves involved in a mental game, a heuristic fiction.”

(Schulze 2003 p. 63)

The boundary between linear and nonlinear structure is perhaps best explored through the comparison of two works from opposing sides of the “border”. Béla Bartók’s Az éjszaka zenéje (The Night’s Music) from the Szabadban (Out of Doors 1926) and Olivier Messiaen’s Regard de l’Onction Terrible (Contemplation of the Awesome Anointing) from Vingt regards sur l’Enfant-Jésus (Twenty Contemplations of the Christ Child 1944) are piano works, of similar length, that share an “arch” (ABCBA) formal structure. Despite the external formal similarities between the works, the composers’ approach to the musical materials comprising the sections is varied.

<table>
<thead>
<tr>
<th>Event</th>
<th>Texture</th>
<th>Dynamic</th>
<th>Articulation</th>
<th>Orchestration</th>
<th>Pulse</th>
<th>Clef</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Contrapuntal</td>
<td>rapid changes (p-p)</td>
<td>legato with some accents and staccatos</td>
<td>Fixed</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>Extended techniques</td>
<td>rapid changes (pp-ff)</td>
<td>Glissandi and other graphical indications</td>
<td>Free</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>Chords</td>
<td>none</td>
<td>none</td>
<td>Fixed</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>Solo</td>
<td>none</td>
<td>Some glissandi, accents and staccatos</td>
<td>Free</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 1. The parametrical variety in Events 1, 2, 3 and 4 of Earle Brown’s Event Synergy II (1967)

The Night’s Music, Bartók’s first excursion into what would become one of his signature musical styles (Curcio 2009 p. 64), can be divided formally into five sections: the outer two eponymous exemplars of nocturnal “nature sounds and noises”, the second and fourth usually referred to as a “Chorale” or “song of loneliness” and a central section featuring a synthetic “peasant flute” melody. The opening phrase of each section is given in Table 2. Although no attempt here is made at a thorough analysis, it is clear even from the fragments presented in the Table that there are relationships between the five sections – if only in the complimentary exploration intervalllic shapes at the head of each melody.

Figure 5: A schematic representation a notional realisation of Earle Brown: Event – Synergy II (1967).
Somfai, who does provide a thorough analysis of The Night’s Music in his article “Analytical Notes on Bartók’s Piano Year of 1926” (1984), comes to the following conclusion, regarding the growth of the work from the seeds of the “hidden melody” (the central stave of section 1 in Table 2.).

One should mention as an almost reflex presence of the organic thinking in Bartók's composition that the hidden melody of the ostinato foreshadows quite clearly the polymodal chromatic line of the third and fourth phrases of the Chorale melody.

(Somfai 1984 p. 10)

Messiaen in contrast, builds Regard de l’Onction Terrible with sections that are entirely distinct and discrete. Healey describes Messiaen’s approach as derived from his exploration of non-retrogradable rhythms — rhythmic palindromes that remain identical when reversed.

The movements in non-retrogradable form demonstrate Messiaen’s application to the structural domain of a technique designed for small-scale use. (...) The eighteenth of the Vingt Regards is a perfect example of the symmetry produced by this form.

(Healey 2008 p. 172)

Non-retrogradable rhythms create peculiarly discrete structures. To paraphrase T. S. Elliot, their “end is in their beginning” and this means their identity is bound to their symmetry and must always terminate in a boundary beyond which they cannot pass.

Gareth Healey identifies palindromic formal structures in a number of Messiaen’s works including movements of the Visions de l’Amen (1943) and Vingt regards sur l’Enfant-Jésus (1944) (Healey 2008 p. 168).

Table 2. The Arch Form structure of Béla Bartók’s Az éjszaka zenéje (The Night’s Music) from Szabadban (Out of Doors 1926)

<table>
<thead>
<tr>
<th>SECTION A</th>
<th>bars 1-17</th>
<th>“nature sounds and noises”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lento</td>
<td>(J +72 - 89)</td>
<td></td>
</tr>
<tr>
<td>m. s. pp</td>
<td>(cort Bk.)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SECTION B</th>
<th>bars 18-37</th>
<th>“Chorale”, “song of loneliness”</th>
</tr>
</thead>
<tbody>
<tr>
<td>andante</td>
<td>(J+60)</td>
<td></td>
</tr>
<tr>
<td>m. s.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SECTION C</th>
<th>bars 38-48</th>
<th>“peasant flute”</th>
</tr>
</thead>
<tbody>
<tr>
<td>m. d.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Sound Scripts: Proceedings of the 2009 Totally Huge New Music Conference, vol. 3 (2011))
The three sections vary in the range of pitch classes that they employ and their compass. The first and fifth sections are fully chromatic and cover seven octaves from A0 to A7. They consist entirely of 016 trichords that chromatically traverse a range of two and a half-octaves. These superimposed chromatic scales employ “chromatic durations” - one descending with durations of increasing length, and the other ascending with durations of decreasing length. The two scales are retrogrades of one another (transposed by a tritone) and the fifth section is also a retrograde of first. The second and fourth sections are based on Messiaen’s fifth mode of limited transposition and have a range of slightly more than seven octaves from Bb0 to B7. They comprise two types of material: melodies of five-note “quintal” chords alternating with a variety of arpeggiation that Healey refers to as “rocket-like groups” (Healey 2008 p. 168). The third section is based on Messiaen’s “Theme of Chords”, seven to ten-note chords of equal duration, and occupies less than 3 octaves from C#2 to B4. The section is fully chromatic, including all twelve notes of the chromatic scale.

Both the Bartók and Messiaen examples are highly determinate and also readily compressible into quite simple formal schemes. It is feasible to envisage other possible avenues for contextualizing these works: they both, for example, exhibit programmatic aspects that might lead to slightly different conclusions about the relationships between their substructures. However the discrete nature of the

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**SECTION B**
bars 49-60
“Chorale”, “song of loneliness”

**SECTION A**
bars 61-65
“nature sounds and noises”

---

Figure 6. Non-retrogradable rhythms from Olivier Messiaen’s *Quartet for the End of Time* (1941). Palindromic durational sets establish their own boundaries through symmetry, because of the finality of the conclusion of the mirrored set of durations.
three kinds of musical materials found in Regard de l’Onction Terrible, resists attempts on a purely musical level to assign an overarching continuity or contingency in the formal structure of the work. For this reason, and in comparison to the Bartok example, Regard de l’Onction Terrible can be considered to have a nonlinear formal structure. These two works occupy opposing sides of the border between linear and nonlinear structure.

Table 3. The Arch Form structure of Olivier Messiaen’s Regard de l’Onction Terrible (Contemplation of the Awesome Anointing) from Vingt regards sur l’Enfant-Jésus (Twenty Contemplations of the Christ Child 1944)

| SECTION A | “chromatic durations” | bars 1-23 |
| SECTION B | “rocket-like groups” | bars 24-90 |
| SECTION C | “chorale” | bars 91-97 |
| SECTION B | “rocket-like groups” | bars 98-177 |
| SECTION A | “chromatic durations” | bars 178-196 |
Vingt regards sur l’Enfant-Jésus (Twenty Contemplations of the Christ Child 1944)

Conclusion
This discussion suggests that a structure of a musical work may be evaluated as Nonlinear on the basis of its degree of integration, contingency, compressibility and determinacy. Results of such an evaluation would be expected to form a continuum of states of structural complexity. Future research might productively focus on confirming these claims through perceptual studies or analysis using tools such as the spectrogram to appraise discontinuity in key form bearing parameters such as pitch, timbre, dynamics and timbre.

Bibliography

Notes


Tonal Harmony, developed over several centuries, make it “particularly susceptible to such reorderings” (Kramer, 1981 p. 545). An online Minuet generator using the Musikalisches Würfelspiel to realise Minuets can be found at http://sunsite.univie.ac.at/Mozart/dice/.

iv McAdams proposes the parameters “timbral brightness, pitch, duration, dynamics and spatial location” as possessing form-bearing capacities (McAdams 1989 p. 195). The potential for referential and/or narrative musical materials to bear form will be discussed below.

v “Musical form is constituted through the division of the musical timespan into sections of a certain size; that the individuality of these sections is brought about through a balance between change and continuity; and that this play of variation inside a frame of overall unity is grounded on the tendency of the human mind to create coherence in event structure” (Kuhl and Jensen 2007 p. 266) also see Snyder (2000) p. 194.


viii These descriptors are used by Somfai (1984 p. 5) and Tallián (1981 p. 144).

ix For an account of Messiaen’s development of the “chromatic durations” technique see Sholl (2008) p. 73.

x Rogosin (1996) p. 119

xi In the preface to Vingt regards sur l’Enfant-Jésus, Messiaen notes three recurring themes that appear in various movements of the work: the “Theme of God”, the “Theme of the Star and the Cross” and the “Theme of Chords”. See Messiaen (1947) p. ix